Effects of Geometer’s Sketchpad on Algebraic Reasoning Competency amongst Students in Malaysia

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Abstract: This study investigates the impact of incorporating Geometer's Sketchpad into GSP-based instruction on the algebraic reasoning proficiency of students. The research outlines the application of GSP-based instruction, utilizing Geometer's Sketchpad in Mathematics education, with a specific focus on solving quadratic problems. A quasi-experimental design, involving non-equivalent pre-test and post-test assessments, was conducted on 60 second-grade students at a private secondary school in Kuala Lumpur. The experimental group, consisting of 30 students, utilized Geometer's Sketchpad in their learning, while the control group, comprising 30 students, received traditional instruction. Data analysis was performed using ANATES 4 and SPSS 25.0 software, incorporating inferential statistics such as paired t-tests and one-way ANCOVA for quantitative data analysis. Both groups underwent an initial pre-test assessment. The research findings reveal a significant disparity in algebraic reasoning proficiency between the two groups, indicating substantial improvement after the intervention. Consequently, the integration of Geometer's Sketchpad in GSP-based instruction contributed to the enhancement of students' algebraic reasoning skills, particularly in quadratic problem-solving. In conclusion, this study underscores the potential of Geometer's Sketchpad as an instructional intervention, urging mathematics educators to contemplate its incorporation as an alternative teaching tool.

A. Introduction

The Malaysian education blueprint has recognized the significance of Science, Technology, Engineering, and Mathematics (STEM) in cultivating and improving logical and analytical thinking amongst students by integrating technology (Ng, 2017). The STEM program adopts the latest technologies as learning platforms to connect students with real-world context. These platforms help them solve real-life problems and explore the opportunity for analysis, evaluation, and innovation. Contextual learning is a potential approach to teaching and learning Mathematics. It indirectly strengthens mathematics reasoning amongst students in real-world circumstances (Han et al., 2015). STEM education dismantles traditional subject boundaries in schools through the integration of diverse disciplines, promotion of scientific inquiry, and cultivation of innovation to actively involve students in the learning process. The program underscores the importance of incorporating technology into the national school curriculum, contributing to the progress of teaching and learning methodologies in the fields of science, engineering, and mathematics. This emphasis places importance on developing multidisciplinary skills and promoting contextual reasoning. Reasoning is a cognitive process of searching for reasons, beliefs, conclusions, actions, or feelings (Shergill, 2012). This process is critical in thinking or argumentations and serves many purposes, such as convincing accepting a particular claim, solving a problem, or integrating ideas into a coherent whole.

Arora (2016) defined reasoning as a deductive skill to conclude known assumptions together with a set of well-established rules. Deductive reasoning is a skill required in the problem statement to top-down logic by inserting a general rule leading to a guaranteed solution. With this skill, an individual can avoid having incorrect responses based on the given perceptual cues. Thus, one reaches a conclusion using evidence from the premise (Hopkinson et al., 2017). These acquired skills enhance student performance and encourage them to explore and generate new ideas to prepare for workplace hardship, where procedural skills in reasoning are necessary to possess facts and information logically (Carnevale & Smith, 2013). The challenge of developing algebraic reasoning abilities amongst students is their different learning styles in solving mathematical problems. Learning styles can be auditory, kinaesthetic, and visual modes. The algebraic reasoning in the classroom setting should come up with multiple representations of the approach in charts, tables, algebraic symbols, and mathematical models. The multiple representations of the known information enable students to find relevant elements to make a mathematical pattern, link a relationship between quantities, and create conjectures on the relationship to prove it (Indraswari et al., 2018). The general indicators helping students in algebraic reasoning are algebraic objects, algebraic operations (transformation), and language types, such as natural, numerical, iconic, gestures, and symbols.

However, the current educational instruction does not build up this mathematical reasoning ability amongst secondary school students. They are commonly taught to solve problems based on mathematical symbol manipulations. Consequently, mathematical reasoning competency amongst these students is not well developed given the absence of
problem-based learning (Napitupulu et al., 2016). The mathematical procedures are addled understanding derived from mathematical knowledge instead of relational understanding. The development of algebraic reasoning competency is particularly significant to enrich one’s mathematical reasoning competency (Smith & Thompson, 2017). The concept behind the quadratic equation is the quadratic function. It can make the quadratic equation characteristics difficult for students to understand regardless of the solution (López et al., 2016). Those who do not have absolute exposure to the historical reflections and knowledge of the development process in solving quadratic equations may experience various misconceptions. Student difficulties in solving quadratic equations are rooted in these misconceptions. The formal definition of the quadratic equation is not clearly shown in students’ conceptual image of quadratic equations; instead, it is mostly influenced by the factors in symbolic manipulation (Kabar & Gözde, 2018). Algebraic reasoning difficulties in solving quadratic equations occur not only in symbolic problems but also in word problems. In word problems, students face challenges in understanding the context and formulating the quadratic equations (Didis & Erbas, 2015). Therefore, students’ perception of the quadratic equation features requires further investigation. In this way, the meanings that they infer from these features and the possible approaches—either conducive or obstructive—to solving actions for quadratic equations will be determined.

Technological tools, such as dynamic mathematical software, can enhance student understanding. The transparency and visualization of algebraic reasoning can be improved in the provided working margin and answer-checking feedback device created to support users in problem-solving and collaborative creative reasoning (Granberg & Olsson, 2015). In a diverse learning environment, students commonly engage in trial-and-error strategies. Hence, teachers facilitate the session by providing solutions for critical situations encountered during problem-solving activities (Arbain & Shukor, 2015). In a technological environment, students are experiencing advanced learning through automated deduction tools in the computer algebra system, such as Geometer’s Sketchpad. Hence, conventional reasoning is no longer relevant. The utilization of Geometer’s Sketchpad software has the potential to enhance students’ proficiency in algebraic reasoning, as noted by Troup (2019). Additionally, Choi (2018) highlights that Geometer’s Sketchpad contributes to the improvement of mathematical reasoning skills by providing assistance in solving intricate geometry problems.

This study seeks to evaluate the effectiveness of integrating the Geometer’s Sketchpad software in improving algebraic reasoning skills in the context of teaching and learning. The specific focus is on solving problems related to quadratic equations with a single variable, a topic covered in the Mathematics curriculum for lower secondary schools in Malaysia, including those in the private sector. As previously mentioned, the use of technological tools has the potential to influence students’ algebraic reasoning. Therefore, this research investigates the impact of utilizing a specific technological tool, the Geometer’s Sketchpad software, on students’ algebraic reasoning abilities. Algebraic reasoning involves various aspects, such as (1) creating conjectures, justifications, generalizations, explanations,
and establishing relationships between numerical quantities, (2) connecting current and past knowledge and facilitating communication between arithmetic and algebra, (3) performing calculations and procedures, and (4) examining and (5) explaining solutions. In summary, this study aims to examine how the Geometer's Sketchpad influences the development of algebraic reasoning skills among second-year high school students.

B. Method

Research Design

In this investigation, a quasi-experimental approach with non-equivalent groups and pre-test/post-test designs, as outlined by Creswell (2012), was employed. Participants were segregated into an experimental group and a control group, and subsequent analysis was conducted based on the collected data. The control group underwent conventional classroom instruction involving lectures, textbooks, and activity books, with no utilization of the Geometer's Sketchpad software. In contrast, the experimental group engaged in quadratic equation tasks that necessitated the use of the Geometer's Sketchpad software to facilitate algebraic reasoning.

Table 1. Pre-test and Post-Test Control Group Design

<table>
<thead>
<tr>
<th>Experiment class</th>
<th>O₁</th>
<th>X₁</th>
<th>O₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control class</td>
<td>O₁</td>
<td>X₂</td>
<td>O₂</td>
</tr>
</tbody>
</table>

Notes:

O₁: Pre-test (algebraic reasoning abilities amongst students)
O₂: Post-test (algebraic reasoning abilities amongst students)
X₁: Geometer’s Sketchpad
X₂: Traditional Approach

The participants in both groups were subjected to a pre-test and a post-test. The experiment group used Geometer’s Sketchpad in their Mathematics class, whereas the control group continued their lessons conventionally. The post-test was given two months after the treatment. As suggested by Campbell & Stanley (1963), the appropriate period is a year after the pre-test. Before conducting the actual study, the subject teachers were properly trained by the researchers for one month with four meetings. This training was held to ensure that the subject teachers would understand the implementation of Geometer’s Sketchpad.

Ethical Approval

The research procedures were read out to all participants. The agreement describes research goals, procedures, and participation in this study. To ensure confidentiality, all answers are anonymous, and no personally identifiable data is collected. This study was approved by the Faculty of Education, University of Malaya (UM.P/PP(IT)/644).
Participants

The experiment involved 60 second-year students of a private Chinese high school in Kuala Lumpur. Both groups had 30 students each. The participants were selected using the convenience sampling method and an intact group. Campbell & Stanley (1963) and Christensen (2001) suggested that sample selection can be achieved based on convenience and intact groups. The convenience sampling method was chosen because the education system in Malaysia places students in fixed classroom settings.

Instruments

The tools utilized for assessing algebraic reasoning were adapted from the framework introduced by Lepak et al. (2018), encompassing five essential competencies. The assessment of algebraic reasoning entails capturing subjective responses related to content on quadratic equations within the broader algebraic subject, showcasing proficiency in algebraic reasoning. To validate the developed instruments, two experts were consulted and briefed on the study's objectives. These experts evaluated concepts and skills, assessed difficulty levels and problem clarity, and gauged the appropriateness of language and terms for second-year high school students. The evaluators confirmed content validity, and adjustments were made based on their feedback. The proposed instrument comprises four constructs assessing learners' abilities in symbolic manipulation, conceptual understanding, analytical skills, and word problem-solving.

The questionnaire was organized into four constructs: the first construct included ten questions, the second and third constructs each featured three questions, and the last section comprised five questions. Throughout the study, students from both groups were required to respond to 21 items in both the pre-test and post-test. Each item was scored based on the competency level suggested by Lepak et al. (2018), with each level receiving one mark. The maximum score for the five competencies in algebraic reasoning is five. The first and second constructs measured algebraic reasoning up to the third level, while the third construct assessed competency up to the fourth level. The final construct included word problems testing algebraic reasoning up to the fifth level. The reliability of the algebraic reasoning test was assessed using Cronbach's alpha in ANATES 4, resulting in a Cronbach's alpha value of 0.780. An instrument is considered reliable when the Cronbach's alpha value exceeds 0.70. However, difficulty and discriminant index results above 30% for each item are deemed acceptable (To, 1996).

Data Analysis

The obtained data was further analyzed quantitatively using ANATES 4 and SPSS 25.0. ANATES 4 was used to determine the discriminant and difficulty index of the instruments. The paired sample t-test was carried out to distinguish any significant difference in students’ competencies on the algebra topic. A one-way ANCOVA test was also conducted to determine the differences between the experiment and control groups.
The findings were controlled by the pre-test. Christensen (2001) noted that ANCOVA is deemed to be a suitable analysis method for quasi-experimental studies with the type of non-randomized control group with a pre-test–post-test design. Given the characteristics of the quasi-experimental study, the participants in both groups were not selected randomly. The homogeneity test was performed to determine the diversity in the intelligence profiles and the proficiency level before the treatment.

C. Result and Discussion

Result

The difference in the pre-test and post-test results of algebraic reasoning competency amongst the students in the experiment group

Before conducting hypothesis testing, an assessment for normality assumption was performed. The Shapiro-Wilk test produced statistics of 0.971 and 0.932. For the experimental group, the pre-test and post-test generated p-values of 0.564 and 0.056, respectively, both surpassing the threshold of 0.05. As a result, there was no violation of the normality assumption for the experiment group in both the pre-test and post-test.

Table 2. Paired Sample T-test Results for the Experiment Group

<table>
<thead>
<tr>
<th>Test</th>
<th>N</th>
<th>M</th>
<th>SD</th>
<th>t</th>
<th>df</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre</td>
<td>30</td>
<td>21.57</td>
<td>5.33</td>
<td>18.081</td>
<td>29</td>
<td>0.001</td>
</tr>
<tr>
<td>Post</td>
<td>30</td>
<td>40.90</td>
<td>5.17</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2 presents the outcomes of the paired sample t-test, depicting the students' competency levels in the experiment group during the pre-test and post-test. The pre-test results (N = 30, M = 21.57, and SD = 5.33) and the post-test results (N = 30, M = 40.90, and SD = 5.17) yielded a t-value of 18.081, with 29 degrees of freedom and a p-value below 0.05. Consequently, the null hypothesis was rejected, indicating a significant difference in the experiment group's competency levels between the pre-test and post-test. The mean score for the pre-test (21.57) surpassed that of the post-test (40.90), suggesting that, at a 5% significance level, students in the experiment group demonstrated higher algebraic reasoning competency following the intervention period. The implementation of Geometer's Sketchpad in GSP-based instruction for solving quadratic problems contributed to the enhancement of algebraic reasoning skills among students. This discovery aligns with a previous study (Ramli et al., 2014), which concluded that GSP plays a pivotal role in aiding students in learning algebra, leading to a substantial improvement in performance after GSP implementation. Similar outcomes were observed in the post-test mean score, indicating a twofold increase in students’ achievement following the GSP-based intervention.

In this study, the Geometer’s Sketchpad was applied to the quadratic problem-solving topic. Amongst the 30 students in the experiment group, 27 obtained extremely high levels of algebraic reasoning competency. The remaining three students also exhibited a high level of improvement, and none remained categorized in the low or medium level of
algebraic reasoning competency after the intervention. A prior study (Kustiawati et al., 2019) also reported similar results after employing an instructional intervention using the dynamic geometry software. Specifically, students who previously ranged at a low or moderate level of mathematical knowledge had significantly improved their mathematical reasoning abilities for problem-solving learning. They also revealed that students in the experiment group had high acquisition and could build a relationship between representations and algebraic operations and formulas and had improved success in the interpretation of the solutions (Övez, 2018).

The difference in pre-test and post-test results of algebraic reasoning competency amongst students in the control group

Prior to conducting hypothesis testing, the normality assumption was assessed. The Shapiro-Wilk test results revealed statistics of 0.965 and 0.937. In the case of the control group, the p-values for the pre-test and post-test were 0.405 and 0.076, respectively, both exceeding 0.05. Consequently, the normality assumption for the control group in both the pre-test and post-test was upheld.

Table 3. Paired Sample T-test Results for the Control Group

<table>
<thead>
<tr>
<th>Test</th>
<th>N</th>
<th>M</th>
<th>SD</th>
<th>T</th>
<th>df</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre</td>
<td>30</td>
<td>16.47</td>
<td>8.01</td>
<td>10.308</td>
<td>29</td>
<td>0.001</td>
</tr>
<tr>
<td>Post</td>
<td>30</td>
<td>32.47</td>
<td>7.27</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3 displays the outcomes of the paired sample t-test, illustrating the competency levels of students in the control group during the pre-test and post-test. The pre-test results (N = 30, M = 16.47, and SD = 8.01) and post-test results (N = 30, M = 32.47, and SD = 7.27) yielded a t-value of 10.308, with 29 degrees of freedom and a p-value of 0.001, which fell below 0.05. Consequently, the null hypothesis was rejected, signifying a noteworthy difference in the control group's competency between the pre-test and post-test. The mean score for the post-test (32.47) exceeded that of the pre-test (16.47), suggesting that the control group exhibited higher algebraic reasoning competency after the intervention period. Specifically, they demonstrated a substantially elevated level of algebraic reasoning competency, with a significance level of 5%, following the intervention period. In summary, traditional instruction also led to an improvement in students' algebraic reasoning competency.

The above findings were consistent with a prior study (Abdullah et al., 2015). They showed that students also exhibited improved mathematical reasoning after the intervention period without using Geometer’s Sketchpad. However, Tutkun & Ozturk (2013) argued that the findings of higher achievement scores do not have any significant relation to mathematical thinking and reasoning level. In the current study, 23 out of 30 students acquired a higher level of algebraic reasoning competency, although they learned the topic using a traditional approach. Nevertheless, three students in the control group still
obtained a low level of algebraic reasoning competency. This finding might be due to the mismatched mathematics level, which could be a result of other external factors, including the teacher, instructional materials, and content, which unintentionally influenced the control group to perform better in the post-test. Hence, the effect of traditional instruction on student performance had no concrete evidence. No clear proof was also found to support that the students in the control group who recorded a low level of algebraic reasoning competency were subjected to traditional intervention. This condition was explained by Khalil et al (2018) that GSP-based learning was an influential factor in favor of low achievers in the treatment group than those in the control group.

The difference in pre-test and post-test results of algebraic reasoning competency amongst students in the control group

Prior to hypothesis testing, an assessment of variance homogeneity was conducted under the assumption of normality. For the treatment group, the Shapiro–Wilk test results showed statistics of 0.971 and 0.932. In the pre-test and post-test, the treatment group obtained p-values of 0.564 and 0.056, respectively. Both groups, including the control group, recorded statistics of 0.965 and 0.937. The control group had p-values of 0.405 and 0.076 on the pre-test and post-test, respectively, exceeding 0.05. Consequently, the normality assumption for both the treatment and control groups in both the pre-test and post-test was affirmed. The F test result and the associated p-value for Levene’s test on the equality of variances showed an F value of 5.155 (df1 = 1, df2 = 58) with a p-value of 0.057, which was greater than 0.05. Accordingly, the assumption of the equality of variances between the experiment and control groups was accepted. The one-way ANCOVA was tested to determine whether the independent variable (instructional intervention) had influenced the dependent variable (post-test) after excluding the initial difference in algebraic reasoning competency level amongst the participants. The ANCOVA test concluded the post-test score for the dependent variable of algebraic reasoning competency. The between-subject factor was instructional groups, and the covariate was the pre-test algebraic reasoning competency score.

<table>
<thead>
<tr>
<th>Source</th>
<th>Mean Square</th>
<th>df</th>
<th>F</th>
<th>Sig.</th>
<th>Partial Eta Squared</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-test</td>
<td>335.808</td>
<td>1</td>
<td>9.715</td>
<td>0.003</td>
<td>0.146</td>
</tr>
<tr>
<td>Group</td>
<td>575.466</td>
<td>1</td>
<td>16.648</td>
<td>0.001</td>
<td>0.226</td>
</tr>
<tr>
<td>Error</td>
<td>34.568</td>
<td>57</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The ANCOVA test results were as follows: F(1, 57) = 16.648, sig = 0.001 (p < 0.05) and $\eta^2_p = 0.226$, with a p-value of less than 0.05. These values showed a significant difference in mean scores for the algebraic reasoning competency levels between the experiment and control groups after the instructional intervention. The mean score was 39.998 for the experiment group and 33.369 for the control group. Hence, the one-way ANCOVA test
Results indicated that the GSP-based instruction had significantly improved algebraic reasoning competency test scores amongst students in the experiment group compared with those in the control group for the quadratic problem-solving topic. The effect was a 5% level of significance, with the pre-test score as the controlled result.

**Table 5. Results of the Estimated Marginal Means for the ANCOVA Test**

<table>
<thead>
<tr>
<th>Group</th>
<th>Mean</th>
<th>Std. Error</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Lower Bound</td>
</tr>
<tr>
<td>Experiment</td>
<td>39.998a</td>
<td>1.112</td>
<td>37.772</td>
</tr>
<tr>
<td>Control</td>
<td>33.369a</td>
<td>1.112</td>
<td>31.143</td>
</tr>
</tbody>
</table>

**Discussion**

The above findings were in agreement with a prior study (Ames & Alon, 2013). They found that the difference between groups was significant when their pre-test scores were controlled. They also concluded that the GSP-based teaching and learning programs enhanced students’ thinking skills and learning outcomes with high inductive reasoning and conceptual knowledge. Similarly, Iji et al (2018) revealed that students who were taught with GSP-based instruction had a more significant effect on their post-test achievement than traditional instruction when the pre-test score was controlled. They also reported that students had greater learning interest in the GSP-based approach than in the traditional approach. Hence, they recommended using Geometer’s Sketchpad in teaching other mathematical concepts, such as algebra.

The current study can be beneficial to Mathematics teachers, students, lecturers, and curriculum developers. These parties are encouraged to use the technological approach, particularly the Geometer’s Sketchpad software, in classroom activities. The outcomes of this research will help teachers to plan appropriate strategies for developing algebraic reasoning competency amongst students and encourage the use of Geometer’s Sketchpad amongst students for either individual or collaborative learning modes. The research findings can serve as a source of perspective to train pre-service teachers to be more innovative and effective in using technology in teaching. Teachers will practice using computer software in advance and acquire pedagogical knowledge to apply the best approach to teach students in this era of industrial technology 4.0. The Malaysian education blueprint 2015–2025 has laid out the significance of cognitive development and thinking skills. Hence, these skills can be accelerated through the integrated implementation of the STEM program. This study is one of the first to promote the Geometer’s Sketchpad software as an advanced teaching tool applicable to all secondary schools in Malaysia. It provides a solid path for future studies, which can examine the subject and the software in a broader view to determine its other effects on students.
D. Conclusion

The results of this study show that students’ competency in algebraic reasoning on quadratic problem solving would change positively after the introduction of Geometer’s Sketchpad as a GSP-based instructional intervention. Although students in the control group also improved in the post-test, the mean difference was not higher than the mean recorded by students in the experiment group. Both groups showed distinct competency enhancement post-intervention. Therefore, this study proves the potential and positive effects of using Geometer’s Sketchpad in the GSP-based instruction for the quadratic problem-solving topic. The scientific evidence of competency improvement amongst students in the experiment group shows the usefulness and effectiveness of the Geometer’s Sketchpad to guide students in learning quadratic problem-solving compared with the traditional teaching approach. The interactive mathematics software allowed students to engage in fun learning and develop mathematical thinking and reasoning. However, the current institutional evaluations on critical thinking in problem-solving were only performed quantitatively by the secondary school teachers.

Acknowledgment

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